

Flavor changing nucleon decay

Nobuhiro Maekawa

*Kobayashi Maskawa Institute, Nagoya University;
Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

Yu Muramatsu

School of Physics, KIAS, Seoul 130-722, Korea

Recent discovery of neutrino large mixings implies the large mixings in the diagonalizing matrices of $\bar{\mathbf{5}}$ fields in $SU(5)$ grand unified theory (GUT), while the diagonalizing matrices of $\mathbf{10}$ fields of $SU(5)$ are expected to have small mixings like Cabibbo-Kobayashi-Maskawa matrix. We calculate the predictions of flavor changing nucleon decays (FCND) in $SU(5)$, $SO(10)$, and E_6 GUT models which have the above features for mixings. We found that FCND can be the main decay mode and play an important role to test GUT models.

PACS numbers:

I. INTRODUCTION

One of the most exciting discoveries in elementary particle physics among the latest 20 years is neutrino oscillation[1, 2], which leads to massive neutrinos and large neutrino mixing angles. Interestingly, this discovery gives an evidence of grand unified theory (GUT)[3], in which unification of forces and unification of quarks and leptons are realized. This evidence for unification of quarks and leptons makes the idea of GUT quite promising, because for unification of forces we have already known an experimental evidence that three gauge couplings meet at a scale, the GUT scale Λ_G [4], especially in supersymmetric (SUSY) GUT[5]. Moreover, the large neutrino mixing angles imply not only large mixing angles of doublet lepton l but also those of right-handed down quark d_R^c in $SU(5)$ GUT because $\bar{\mathbf{5}}$ field of $SU(5)$ contains l and d_R^c . This suggests an interesting possibility that the flavor changing processes are seen in nucleon decay which is the most important prediction of GUT. In this paper, we study the flavor changing nucleon decay and propose that the flavor changing nucleon decay can be a key observation for GUT.

II. QUALITATIVE EVIDENCE FOR $SU(5)$ UNIFICATION

First, we explain the qualitative evidence for the unification of matters. In $SU(5)$ GUT, Yukawa interactions with Higgs fields $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ are given by

$$\begin{aligned} \mathcal{L}_Y = & (Y_u)_{ij} \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H + (Y_{d,e})_{ij} \mathbf{10}_i \bar{\mathbf{5}}_j \bar{\mathbf{5}}_H \\ & + (Y_{\nu_D} M_{\nu_R}^{-1} Y_{\nu_D}^t)_{ij} \bar{\mathbf{5}}_i \bar{\mathbf{5}}_j \mathbf{5}_H \mathbf{5}_H, \end{aligned} \quad (1)$$

where $\mathbf{10}$ fields contain doublet quark q , right-handed up quark u_R^c , and right-handed charged lepton e_R^c , and the last term can be obtained from $(Y_{\nu_D})_{ij} \bar{\mathbf{5}}_i \mathbf{1}_j \mathbf{5}_H + (M_{\nu_R})_{ij} \mathbf{1}_i \mathbf{1}_j$ by integrating the right-handed neutrino fields $\mathbf{1}_i$. Here $i = 1, 2, 3$ is the index for generation, and Y_u , $Y_{d,e}$, Y_{ν_D} , and M_{ν_R} are Yukawa matrix of up type

quarks, that of down type quarks and charged leptons, Dirac neutrino Yukawa matrix, and right-handed neutrino mass matrix, respectively. These unified structures for Yukawa interactions are corresponding to the classification of the hierarchies of the observed quark and lepton masses that up type quark masses have the strongest hierarchy, neutrino masses have the weakest, and down type quark and charged lepton masses have middle hierarchies if neutrino mass hierarchy is normal (not inverted). Moreover, if we assume that $\mathbf{10}$ fields induce stronger hierarchy in Yukawa couplings than $\bar{\mathbf{5}}$ fields, these various hierarchies for quark and lepton masses can be explained. Furthermore, this assumption explains that quark mixings are smaller than lepton mixings at the same time if we use a reasonable expectation that the stronger hierarchy leads to smaller mixings. This brilliant chemistry between the Yukawa structure in $SU(5)$ GUT and the observed hierarchies of quark and lepton masses and mixings is quite non-trivial, and therefore it can be regarded as an experimental signature for unification of quarks and leptons in $SU(5)$ GUT.

III. E_6 UNIFICATION

E_6 GUT[6–8] is more attractive because the assumption in the $SU(5)$ GUT can be derived, and as a result, various Yukawa matrices can be derived from one basic Yukawa hierarchy[8]. The fundamental representation in E_6 is divided into $SO(10)(SU(5))$ representations as

$$\mathbf{27} = \mathbf{16}(\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}) + \mathbf{10}(\mathbf{5} + \bar{\mathbf{5}}') + \mathbf{1}(\mathbf{1}). \quad (2)$$

This $\mathbf{27}$ includes one generation quarks and leptons in addition to one pair of vector-like fields $\mathbf{5} + \bar{\mathbf{5}}$ and a singlet. If we introduce three $\mathbf{27}_i$ ($i = 1, 2, 3$) for three generation quarks and leptons, we have six $\bar{\mathbf{5}}$ fields. Three of six $\bar{\mathbf{5}}$ fields become superheavy after developing the vacuum expectation values (VEVs) of $\mathbf{27}_H$ and $\mathbf{27}_C$ through the Yukawa interactions

$$W_Y = (Y^H)_{ij} \mathbf{27}_i \mathbf{27}_j \mathbf{27}_H + (Y^C)_{ij} \mathbf{27}_i \mathbf{27}_j \mathbf{27}_C, \quad (3)$$

where the VEV of $\mathbf{27}_H$ breaks E_6 into $SO(10)$ and the VEV of $\mathbf{27}_C$ breaks $SO(10)$ into $SU(5)$. Once we fix Y^H , Y^C , $\langle \mathbf{27}_H \rangle$, and $\langle \mathbf{27}_C \rangle$, 3×6 mass matrix of three $\mathbf{5}$ s and six $\mathbf{5}$ s is determined, and therefore, three massless modes $\mathbf{5}_i^0$ are fixed. Here we assume that these Yukawa couplings Y^H and Y^C have strong hierarchy corresponding to the hierarchy of $\mathbf{10}$ of $SU(5)$. Typically, we take

$$Y^H \sim Y^C \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (4)$$

where a unit of hierarchy $\lambda \sim 0.22$ is taken to be around the Cabibbo mixing to obtain Cabibbo-Kobayashi-Maskawa (CKM) matrix[9]. The $O(1)$ coefficients are omitted usually in this paper. Then, two $\mathbf{5}$ fields from $\mathbf{27}_3$ become superheavy unless $\langle \mathbf{27}_H \rangle \ll \langle \mathbf{27}_C \rangle$ because they have larger Yukawa couplings and therefore have larger mass parameters. The three massless modes $\mathbf{5}_i^0$ come from the first two generation fields $\mathbf{27}_1$ and $\mathbf{27}_2$ which have smaller Yukawa couplings. As a result, three $\mathbf{5}_i^0$, whose main modes typically become $(\mathbf{5}_1, \mathbf{5}'_1, \mathbf{5}_2)$, induce milder Yukawa hierarchy than $\mathbf{10}_i$ fields, that is nothing but what we assume in the $SU(5)$ GUT to obtain realistic hierarchies of quark and lepton masses and mixings. Note that $\mathbf{5}_2^0 \sim \mathbf{5}'_1 + \lambda^\Delta \mathbf{5}_3$ has Yukawa couplings through the mixing with $\mathbf{5}_3$ when the Higgs $\mathbf{5}_H$ and $\mathbf{5}_H$ are included in $\mathbf{10}_H$ of $SO(10)$ in $\mathbf{27}_H$. Then we can obtain realistic Yukawa hierarchies as

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, Y_d \sim Y_e^t \sim Y_{\nu_D}^t \sim \begin{pmatrix} \lambda^6 & \lambda^{\Delta+3} & \lambda^5 \\ \lambda^5 & \lambda^{\Delta+2} & \lambda^4 \\ \lambda^3 & \lambda^\Delta & \lambda^2 \end{pmatrix}, \quad (5)$$

when $\Delta \sim 2.5$. The right-handed neutrino masses are obtained from

$$\frac{(Y^{XY})_{ij}}{\Lambda} \mathbf{27}_i \mathbf{27}_j \mathbf{27}_X \mathbf{27}_Y, \quad (6)$$

where $X, Y = \bar{H}, \bar{C}$, Λ is the cutoff scale, after developing the VEVs $|\langle \mathbf{27}_H \rangle| = |\langle \mathbf{27}_{\bar{H}} \rangle|$ and $|\langle \mathbf{27}_H \rangle| = |\langle \mathbf{27}_{\bar{H}} \rangle|$. Here we take $Y^{XY} \sim Y^H \sim Y^C$. All quark and lepton mass matrices can be diagonalized by unitary matrices for $\mathbf{10}$ fields and $\mathbf{5}$ fields

$$V_{10} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, V_{\mathbf{5}} \sim \begin{pmatrix} 1 & \lambda^{3-\Delta} & \lambda \\ \lambda^{3-\Delta} & 1 & \lambda^{\Delta-2} \\ \lambda & \lambda^{\Delta-2} & 1 \end{pmatrix}, \quad (7)$$

and we can obtain realistic CKM matrix $V_{\text{CKM}} \sim V_{10}$ and the Maki-Nakagawa-Sakata (MNS) matrix[10] $V_{\text{MNS}} \sim V_{\mathbf{5}}$, when $\Delta \sim 2.5$. Note that the important prediction $(V_{\text{MNS}})_{13} \sim (V_{\text{CKM}})_{12}$, which was confirmed by recent neutrino experiments as $(V_{\text{MNS}})_{13} \sim 0.15$ [2], is caused by $\mathbf{5}_3^0 \sim \mathbf{5}_2$. Therefore, to obtain the realistic hierarchies of quark and lepton masses and mixings, it is essential that the $\mathbf{5}'_1$, which comes from $\mathbf{10}$ of $SO(10)$, becomes the second generation $\mathbf{5}$ field $\mathbf{5}_2^0$. That structure is important

to study of the prediction of the nucleon decay in the next section.

Note that the relation $\mathbf{5}_2^0 \sim \mathbf{5}' + \lambda^\Delta \mathbf{5}_3$ can be realized even in $SO(10)$ unification, if $\mathbf{10}$ of $SO(10)$, which provides $\mathbf{5}'$, is introduced as a matter field[11]. Therefore, we have three GUT models which satisfy the Yukawa hierarchy hypothesis, “ $\mathbf{10}$ fields induce stronger hierarchy in Yukawa couplings than $\mathbf{5}$ fields”. Their unification groups are $SU(5)$, $SO(10)$, and E_6 . Next, we study how to identify these unification group by observing various partial nucleon decay widths.

IV. NUCLEON DECAY

In this paper, we concentrate on the nucleon decay via dimension 6 operators[12], because the nucleon decay via dimension 5 operators[13] is strongly dependent on the explicit model of GUT Higgs sector which is expected to have big modification to solve the most difficult problem called the doublet-triplet splitting problem[14] and because it is strongly suppressed in natural GUT in which the difficult problem is solved with natural assumption[11, 15, 16].

The dimension 6 effective operators which induce nucleon decay in E_6 GUT are produced via mediation by $SU(5)$ superheavy gauge boson X , $SO(10)$ superheavy gauge boson X' , and E_6 superheavy gauge boson X'' as[18]

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{g_G^2}{M_X^2} \{ -(\bar{e}_{Ri}^c u_{Rj})(\bar{q}_j^c q_i) + (\bar{l}_i^c q_j)(\bar{u}_{Rj}^c d_{Ri}) \\ & + (\bar{l}_i^c q_j)(\bar{u}_{Rj}^c D_{Ri}) \} + \frac{g_G^2}{M_{X'}^2} (\bar{l}_i^c q_j)(\bar{u}_{Ri}^c d_{Rj}) \\ & + \frac{g_G^2}{M_{X''}^2} (\bar{l}_i^c q_j)(\bar{u}_{Ri}^c D_{Rj}) \end{aligned} \quad (8)$$

where g_G is the unified gauge coupling and the superheavy gauge boson masses M_X , $M_{X'}$, and $M_{X''}$ are dependent on the VEVs of the GUT Higgs which break E_6 into the SM gauge group. Here, large character denotes $\mathbf{5}'$ field which comes from $\mathbf{10}$ of $SO(10)$. In the $SO(10)$ GUT, we just take $M_{X''} \rightarrow \infty$, and in $SU(5)$ GUT, we take $M_{X'}, M_{X''} \rightarrow \infty$ and neglect the interactions which include the large character fields. Note that the nucleon decay via dimension 6 operators depends on Yukawa couplings, although this is via gauge interactions. The situation is similar to the weak interaction. The weak interaction is also the gauge interaction, but we have CKM mixings which are determined by Yukawa couplings. For the nucleon decay, the nucleon decay via dimension 6 operators depends on the diagonalizing matrices for Yukawa matrices. However, we have already understood the mixings in GUT as the qualitative evidence for the $SU(5)$ GUT. Especially for the diagonalizing matrices, V_{10} and $V_{\mathbf{5}}$ are fixed as CKM matrix and MNS matrix, respectively, except $O(1)$ coefficients. Therefore, these ambiguities are almost fixed by our understanding of Yukawa

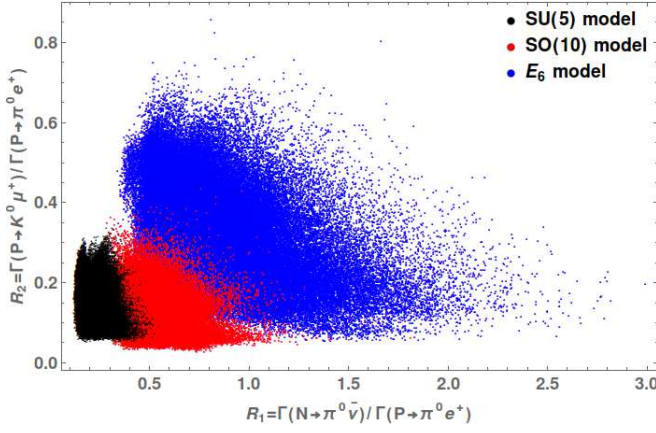


FIG. 1: The distribution of 10^5 model points for $SU(5)$ (black), $SO(10)$ (red), and E_6 (blue) GUTs with horizontal axis $R_1 = \Gamma(N \rightarrow \pi^0 \bar{\nu}) / \Gamma(P \rightarrow \pi^0 e^+)$ and vertical axis $R_2 = \Gamma(P \rightarrow K^0 \mu^+) / \Gamma(P \rightarrow \pi^0 e^+)$. The superheavy gauge boson masses are taken to be $M_X = M_{X'} = \sqrt{2}M_{X''}$.

structures. Therefore, we can compare the predictions of nucleon decays in $SU(5)$, $SO(10)$ and E_6 GUTs.

Important observation to find useful nucleon decay modes for identification of unification group is that all four fermions in the first term in Eq. (8) come from **10** of $SU(5)$ fields, and in the other terms two of four fermions come from **5** fields. Since X' and X'' gauge interactions induce only the effective interactions with **5** fields, we should look for the nucleon decay modes in which the operators with **5** fields are significant to identify the unification group.

Since all operators with **5** fields include a lepton doublet while **10** field includes no neutrino, the modes with neutrino can be important to identify the unification group. The decay mode $N \rightarrow \pi^0 \bar{\nu}$ ¹ has been studied in the literature for the identification[17, 18]. Especially in Ref[18], we have shown that two ratios $R_1 \equiv \Gamma(N \rightarrow \pi^0 \bar{\nu}) / \Gamma(P \rightarrow \pi^0 e^+)$ and $R_2 \equiv \Gamma(P \rightarrow K^0 \mu^+) / \Gamma(P \rightarrow \pi^0 e^+)$ are useful to identify three unification group as in Fig. 1, where we have 10^5 model points for each unification group $SU(5)$ (black points), $SO(10)$ (red points), and E_6 (blue points) and the magnitudes of the $O(1)$ coefficients of diagonalizing matrices are determined randomly between 0.5 and 2. We adopt superheavy gauge boson masses $M_X = M_{X'} = \sqrt{2}M_{X''}$ as in the previous paper[18].² In the calculations in this paper, we

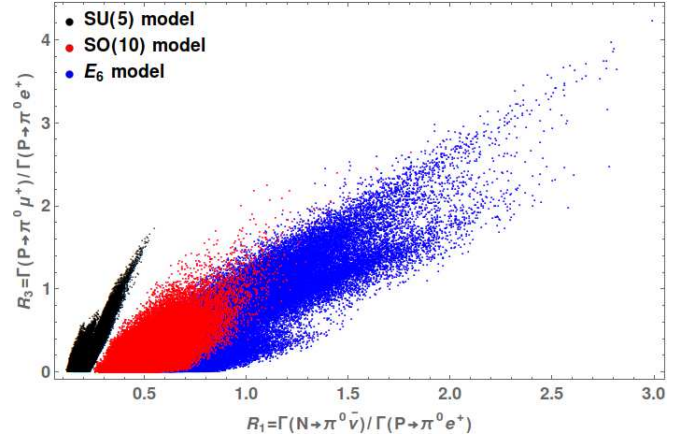


FIG. 2: The distribution of 10^5 model points for $SU(5)$ (black), $SO(10)$ (red), and E_6 (blue) GUTs with horizontal axis $R_1 = \Gamma(N \rightarrow \pi^0 \bar{\nu}) / \Gamma(P \rightarrow \pi^0 e^+)$ and vertical axis $R_3 = \Gamma(P \rightarrow \pi^0 \mu^+) / \Gamma(P \rightarrow \pi^0 e^+)$. The superheavy gauge boson masses are taken to be $M_X = M_{X'} = \sqrt{2}M_{X''}$.

use the hadron matrix elements calculated by lattice[19], and the renormalization factors of the minimal SUSY $SU(5)$ GUT as $A_R = 3.6$ for the operators which include a right-handed charged lepton e_R^c and $A_R = 3.4$ for the operators which include the doublet leptons l as the reference values[20]. The ratio R_2 is sensitive to flavor structure of the second generation, and very useful to identify $SO(10)$ and E_6 unification group. Interestingly, R_1 can be larger than one especially for higher rank unification group like E_6 . Of course the results are strongly dependent on the mass spectrum of superheavy gauge bosons. If $M_{X''} \gg M_{X'} = M_X$, the E_6 model points shrink to $SO(10)$ model points, and when $M_{X'}$ becomes much larger than M_X , the $SO(10)$ model points shrink to the $SU(5)$ model points. However, we can say that if $R_1 > 0.5$, $SU(5)$ is implausible and if $R_1 > 1$, E_6 is preferable. Unfortunately, the detection efficiency for the mode $N \rightarrow \pi^0 \bar{\nu}$ is not so high as $P \rightarrow \pi^0 e^+$ mode[21, 22], and therefore, it requires extremely more powerful experiments to observe the mode $N \rightarrow \pi^0 \bar{\nu}$ even if $R_1 > 1$.

In this paper, we propose novel modes which may be useful for the identification of unification group. Essential point is that **5** fields have large mixings in diagonalizing matrices while **10** fields have small mixings. And therefore, flavor changing nucleon decay, for example, $P \rightarrow \pi^0 \mu^+$ or $P \rightarrow K^0 e^+$, becomes more important for higher rank unification group. In Figs. 2 and 3, we have calculated the two ratios $R_3 \equiv \Gamma(P \rightarrow \pi^0 \mu^+) / \Gamma(P \rightarrow$

¹ In the decay modes which include neutrino, we sum up over the flavor of neutrino because the nucleon decay detectors do not distinguish neutrino types.

² In this paper, we have not fixed $V_{u_R^c} = 1$ (and $V_{d_R^c} = 1$ for $SU(5)$), which are adopted in Ref. [18]. Theoretically we can fix those diagonalizing matrices without loss of generality. If we have not imposed any constraints to the other diagonalizing matrices, it would not produce any changes in the results. However, in our analysis, we constrained the $O(1)$ coefficients of the

other diagonalizing matrices, and therefore, the results depends on whether these conditions are imposed or not. We think that the results without these conditions become similar to the results with these conditions with wider allowed range for the $O(1)$ coefficients. Therefore, distributions of model points have become wider in this paper than in the previous one.

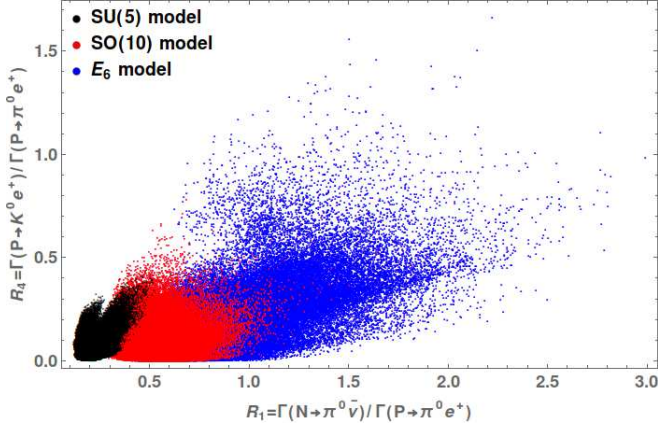


FIG. 3: The distribution of 10^5 model points for $SU(5)$ (black), $SO(10)$ (red), and E_6 (blue) GUTs with horizontal axis $R_1 = \Gamma(N \rightarrow \pi^0 \bar{\nu}) / \Gamma(P \rightarrow \pi^0 e^+)$ and vertical axis $R_4 \equiv \Gamma(P \rightarrow K^0 e^+) / \Gamma(P \rightarrow \pi^0 e^+)$. The superheavy gauge boson masses are taken to be $M_X = M_{X'} = \sqrt{2}M_{X''}$.

$\pi^0 e^+$) and $R_4 \equiv \Gamma(P \rightarrow K^0 e^+) / \Gamma(P \rightarrow \pi^0 e^+)$ with horizontal axis R_1 in 10^5 model points of $SU(5)$ (black), $SO(10)$ (red), and E_6 (blue) GUTs with the superheavy gauge boson masses $M_X = M_{X'} = \sqrt{2}M_{X''}$. Interestingly, the $SU(5)$ model points are clearly separated from $SO(10)$ and E_6 model points in Fig. 2, while Fig. 1 has no such separation. One more interesting point is that there are a lot of model points with $R_3 > 1$. Since the detection efficiency of the $P \rightarrow \pi^0 \mu^+$ is as large as that of $P \rightarrow \pi^0 e^+$ [21], the flavor changing nucleon decay mode $P \rightarrow \pi^0 \mu^+$ can be found earlier than $P \rightarrow \pi^0 e^+$ if $R_3 > 1$. On the contrary, R_4 is comparatively smaller, mainly because the mode $\Gamma(P \rightarrow K^0 e^+)$ has the phase space suppression and smaller hadron matrix elements. Note that there is a tendency to obtain larger R_1 for larger R_3 .

Although it may not be so clear in these figures, GUT with larger rank unification group predicts larger FCND. Actually, it is seen in concrete numbers of model points with $R_3 > 1$ (17% in E_6 , 0.7% in $SO(10)$ and 0.5% in $SU(5)$).

It must be useful to stress the advantage of the neutrino modes like $N \rightarrow \pi^0 \bar{\nu}$ for identification of unification group, although such modes have disadvantage for the detection. The most important feature for $\Gamma(N \rightarrow \pi^0 \bar{\nu})$ is that the value becomes larger for GUT with larger rank unification group, especially when **10** fields have small mixings. Actually, when $V_{10} = 1$, we can show that

$$\frac{\Gamma(N \rightarrow \pi^0 \bar{\nu})}{\Gamma_{SU(5)}(N \rightarrow \pi^0 \bar{\nu})} = 1 + \alpha(2 + \alpha)|V_{d_R^c}|^2 + \beta(2 + \beta)|V_{d_R^c}|^2 \quad (9)$$

where $\alpha \equiv M_X^2 / M_{X'}^2$, and $\beta \equiv M_X^2 / M_{X''}^2$. Here we have summed the flavor of neutrinos, that is important in this calculation. Obviously R_1 becomes larger for larger unification group. This feature is quite important to identify the unification group.

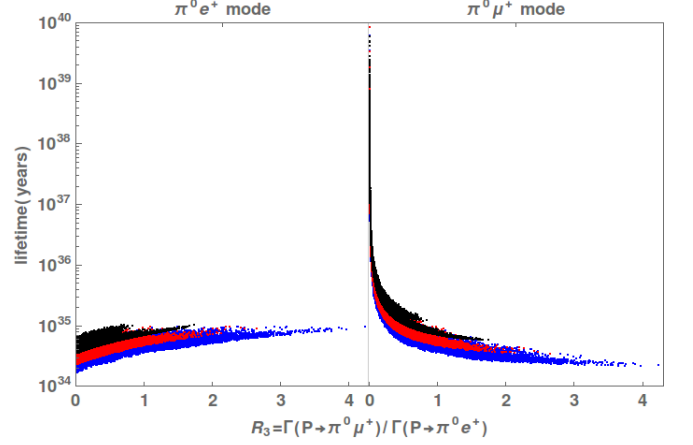


FIG. 4: The distribution of 10^5 model points for $SU(5)$ (black), $SO(10)$ (red), and E_6 (blue) GUTs with $M_X/g_G = 1 \times 10^{16}$ GeV. Horizontal axis is $R_3 = \Gamma(P \rightarrow \pi^0 \mu^+) / \Gamma(P \rightarrow \pi^0 e^+)$ and vertical axis is partial lifetime for $P \rightarrow \pi^0 e^+$ and $P \rightarrow \pi^0 \mu^+$, which are proportional to $(M_X/g_G)^4$. The superheavy gauge boson masses are taken to be $M_X = M_{X'} = \sqrt{2}M_{X''}$.

V. DISCUSSION AND SUMMARY

Recently, two events have been found in the signal region for the process $P \rightarrow \pi^0 \mu^+$ [23], though these are still consistent with the background expected to be 0.9 event mainly from atmospheric neutrino events. If the signature for the flavor changing nucleon decay $P \rightarrow \pi^0 \mu^+$ has been found in SuperKamiokande, higher rank unification group like $SO(10)$ or E_6 is preferable when the mixings of **10** fields are small. The predicted partial lifetime for $M_X/g_G = 1 \times 10^{16}$ GeV is presented in Fig. 4. Obviously, for larger R_3 , longer partial lifetime for $P \rightarrow \pi^0 e^+$ and shorter partial lifetime for $P \rightarrow \pi^0 \mu^+$ are obtained. In $SU(5)$, both partial lifetimes become longer than in $SO(10)$ and E_6 . If the signature is from the real nucleon decay process, it is obvious that the usual MSSM predicted value $M_X/g_G \sim 3 \times 10^{16}$ is too large to explain the events even if the ambiguities in Hadron matrix elements[19] are taken into account. Therefore, to explain the signal, larger unification gauge coupling g_X (it requires extra vector-like fields in addition to the MSSM fields.), and/or smaller superheavy gauge boson mass M_X are required. Note that both features are predicted in the natural GUT [11, 15, 16], in which the nucleon decay via dimension 6 operators is enhanced while that via dimension 5 is suppressed.

Which mode will be found next? We expect that $\Gamma(N \rightarrow \pi^0 \bar{\nu})$ can be larger than $\Gamma(P \rightarrow \pi^0 e^+)$, since R_3 is positively correlated with R_1 as in Fig. 2. However, since the detection efficiency for the mode $N \rightarrow \pi^0 \bar{\nu}$ is much smaller than that for $P \rightarrow \pi^0 e^+$, we can predict that next mode should be $P \rightarrow \pi^0 e^+$. Of course, the other modes, $N \rightarrow \pi^0 \bar{\nu}$ and $P \rightarrow K^0 e^+$, are expected to be found in future experiments like HyperKamiokande

[24]. The observation of these modes is quite important and gives us critical hints for studying GUT models.

In this paper, we have emphasized the importance of flavor changing nucleon decay, whose observation may identify the unification group. Especially, the mode $P \rightarrow \pi^0 \mu^+$ is important because the detection efficiency is as large as the usual mode $P \rightarrow \pi^0 e^+$. The partial lifetime of $P \rightarrow \pi^0 \mu^+$ can be shorter than that of $P \rightarrow \pi^0 e^+$ especially in E_6 GUT. Although most of model points predict longer partial lifetime of $P \rightarrow \pi^0 \mu^+$ than that of $P \rightarrow \pi^0 e^+$, it is important to pay attention to the mode

$P \rightarrow \pi^0 \mu^+$ even if the present signal for $P \rightarrow \pi^0 \mu^+$ is from the back ground processes.

VI. ACKNOWLEDGEMENT

This work is supported in part by National Research Foundation of Korea (NRF) Research Grant NRF-2015R1A2A1A05001869 and Grants-in-Aid for Scientific Research from MEXT of Japan(No. 15K05048).

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